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Scaling relationships between stratabound pressure solution cleavage spacing and layer thickness in a folded carbonate multilayer of the Northern Apennines (Italy)

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ABSTRACT

Pressure solution cleavage is one of the most important deformation structures at shallow crustal levels. Its relationships with environmental, dynamical, textural, and chemical parameters have been broadly studied, particularly at the microscale. However, it is still under debate whether, at the outcrop scale, cleavage surfaces tend to form at rather constant spacing or not, and whether, in the case of stratabound elements, cleavage spacing scales with the host layers thickness.

This work reports on relationships between tectonic pressure solution cleavage spacing (S) and bed thickness (H) from a folded carbonate multilayer of the Northern Apennines. Data were collected mainly in three well-layered carbonatic units, where beds are separated by thin clayish films which acted as barrier for the cleavage vertical propagation, determining its stratabound appearance. Statistical analysis of cleavage spacing and spacing to bed thickness ratio allows recognising a dependence of the cleavage spacing on the host layer thickness. Our analyses also suggest that this dependence relates to an infilling dominated evolution of pressure solution cleavage, where new dissolution surfaces preferentially develop between old cleavages characterised by high spacing to bed thickness ratios.

1. Introduction

Pressure solution cleavage is one of the most important deformation structures occurring in rocks deformed at shallow crustal levels, particularly in carbonate successions exposed both in fold-and-thrust belts (e.g. Alvarez et al., 1978; Mitra and Yonkee, 1985; Marshak and Engelder, 1985; Holl and Anastasio, 1995; Ohlmacher and Aydin, 1995; Sans et al., 2003; Tavani et al., 2006) and in their slightly deformed adjacent foreland sectors (e.g. Arthaud and Mattauer, 1969; Illies, 1975; Railsback and Andrews, 1995). It consists of irregular surfaces coated with residues of insoluble materials (Stockdale, 1922; Dunnington, 1954; Park and Schot, 1968) and usually oriented perpendicular to the maximum acting stress (e.g. Fletcher and Pollard, 1981; Koehn et al., 2007). The relationships between the pressure solution process, the environmental conditions of deformation (e.g. Dieterich, 1969; Carannante and Guzzetta, 1972; Siddans, 1972; Weyl, 1959; Rutter, 1976, 1983; Groshong, 1988; Andrews and Railsback, 1997), and the textural/chemical rock properties (e.g. Marshak and Engelder, 1985; Peacock and Azzam, 2006) have been widely studied, particularly at the microscale. On the other hand, it is still unclear whether, at the outcrop scale, pressure solution development is an organized process, where distinct surfaces tend to develop at rather constant spacing (e.g. Alvarez et al., 1978; Fletcher and Pollard, 1981; Merino et al., 1983; Fueten et al., 2002), or not (e.g. Railsback, 1998), and whether, in the case of stratabound elements, solution cleavage spacing scales with the host layer thickness (e.g. Durney and Kisch, 1994; Tavani et al., 2006, 2008) or not (e.g. Alvarez et al., 1978; Holl and Anastasio, 1995).

With progressing deformation, two processes concur to reduce pressure solution cleavage spacing and, accordingly, to modify the statistical attributes of a given cleavage population: (1) dissolution of material along cleavage surfaces, which implies the reduction of their spacing (e.g. Stockdale, 1922; Dunnington, 1954; Park and Schot, 1968); (2) infilling, which is the process whereby new surfaces form between two pre-existing surfaces (e.g. Merino et al., 1983; Narr and Suppe, 1991; Becker and Gross, 1996; Bai and Pollard, 2000) (Fig. 1a). These processes produce different evolutionary pathways concerning the reduction of the cleavage spacing as deformation progresses with time. If the dominant process is the dissolution of the microlithons, cleavage spacing will reduce progressively with time. On the other hand, if infilling occurs cleavage spacing will follows a stepwise reduction (Fig. 1a). These two end-member processes will result in different cleavage spacing statistical attributes of a cleavage population.

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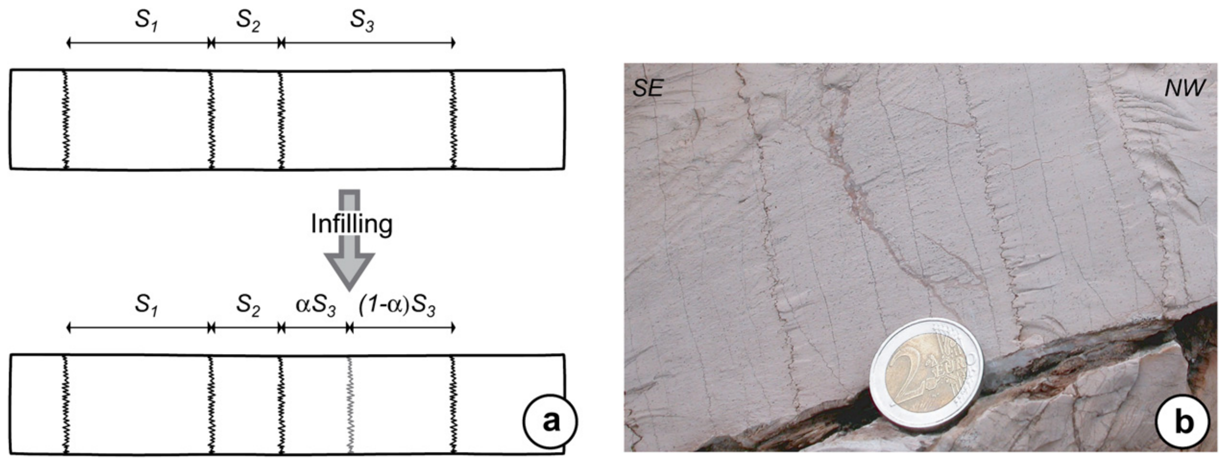

Fig. 1. (a) Cross-sectional spacing variation associated with cleavage infilling. See text for details. (b) Example of cleavage infilling, where at least two distinct cleavage generations are visible. Scaglia Rossa Formation. Northern Apennines (Italy).

In some circumstances it may be possible to infer infilling has occurred from field observations as we may expect that the newly formed cleavage domains will be thinner and smoother or less stylolitic (Fig. 1b). However, a question arises if, not having field evidences, the statistical analysis of cleavage populations may distinguish which of these end-member processes (acting together during deformation progression) plays the major role during cleavage evolution.

In order to conduct such analysis, it is important to collect data in outcrops characterised by different deformation intensities and to ensure constancy, in different sampled populations, of the mechanical/environmental parameters conditioning the cleavage spacing (like rock composition, temperature, fluid circulation; e.g. Marshak and Engelder, 1985).

In practice, it is rather difficult to reduce the "lithological bias", particularly when the area of investigation is bigger than few km^2. This bias can be at least partially reduced by: (1) increasing the number of observations; (2) analysing and comparing data collected in outcrops characterised by rather similar mechanical/environmental conditions. When satisfied, these conditions allow the assumption that the mechanical/environmental parameter variability is low and similar in the different sampled populations, which in turn allows evaluation of whether, in a first approximation, a given evolutionary pathway is consistent with data.

The aim of this work is to find a procedure to unravel the predominant processes acting during pressure solution cleavage development and spacing reduction as deformation progresses. To check the proposed method, a cleavage data set has been collected from a folded carbonate multilayer in the Sibillini anticline of the Northern Apennines.

2. Cleavage spacing theoretical models

2.1. Spacing reduction by infilling

When a cleavage population P_i is "infilled" it transforms to P_j (see Table 1 for *full definition of parameters*). Each infilled cleavage is characterised by a m_{in} value (being m_{in} either S_{in} or S_{in}/H_{in}) in the P_i population that is replaced, in the P_j population, by $m' = \alpha_{in} m_{in}$ and $m'' = (1-\alpha_{in}) m_{in}$ $(0 \le \alpha_{in} \le 1)$ (Fig. 1). In a cross-sectional view, this process includes both apparent infilling, if the infilling is due to the lateral propagation of "old" cleavages, and infilling *sensu strictu* if the infilling relates with the development or lateral propagation of newly developed cleavages.

The average values of m (i.e. $\overline{m}$) in P_j relates to that of P_i through the following equation (see Appendix for derivation):

$$\overline{m}_{\mathrm{j}} = \overline{m}_{\mathrm{i}} \frac{n}{n+f} \tag{1}$$

where n is the data number in the P_i population and f is the number of "infilled" cleavages (so that the number of cleavages in the P_j population is $n+f$)

In a continuum space, the variance of m (σ^2) in a given population relates to its average value ($\overline{m}$) through the following relationship:

$$\frac{\partial \sigma^2}{\partial \overline{m}} = \frac{\sigma^2}{\overline{m}} + \frac{+2\left(\sum_{\mathrm{in}=1}^{f} \alpha_{\mathrm{in}}(1-\alpha_{\mathrm{in}}) m_{\mathrm{in}}^2\right) \Big/ f - (\overline{m})^2}{\overline{m}} \tag{2}$$

This equation can be simplified if we assume a constant

Table 1
Definition of variables used in Sections 3, 4 and in Appendix.

S	Distance between adjacent sub-parallel cleavages measured perpendicular to the cleavage surfaces
H	Layer thickness
P_i	The cleavage population in the ith step of deformation
m	The equations describing the evolution of cleavage spacing during the infilling process are identical to those describing the evolution of cleavage S/H. In order to avoid duplicating each equation (one for S and one for S/H) we introduce the parameter m that, accordingly, in the following equations can be read as either cleavage spacing or cleavage S/H
m_{in}	The m value of the cleavage that, in the next step of deformation, will be infilled.
m' and m''	The m value of the two cleavages that replace m_{in} after infilling (see Fig. 1). By definition the sum of m' and m'' is equal to m_{in}, so that m' and m'' can also be expressed as: $m' = \alpha_{\mathrm{in}} m_{\mathrm{in}}$; $m'' = (1-\alpha_{in}) m_{in}$, being $0 \le \alpha_{in} \le 1$
$\overline{m}_i$	Average value of m in the P_i population
n	Number of individuals in a given P population
f	Number of cleavage that will be infilled in the next step of deformation, so that, being n the number of cleavage in the P_i population, the number of cleavages in the P_j population (which results from the deformation by infilling of P_i) is $n+f$
Q	Proportionality factor between the original population and the infilled part of the dataset. This parameter is introduced by assuming that such a relationship exists, so that the mathematical handling of equation (2) can be extremely simplified
ε_{ij}	Amount of cleavage perpendicular strain necessary to transform a population P_i into P_j in the hypothesis that spacing reduction entirely occurs due to removal of material along the cleavage surface

relationship between the cleavage population and its infilled part, and we introduce a parameter (Q) given by the following equation:

$$Q = \frac{2\left(\sum_{in=1}^{f}\alpha_{in}(1-\alpha_{in})m_{in}^2\right)}{f} \Big/ \frac{\sum_{l=1}^{n} m_l^2}{n} \qquad (3)$$

In this case, the solution for equation (2) is provided by:

$$k(\overline{m})^{Q+1} = \sigma^2 + (\overline{m})^2 \qquad (4)$$

In this power-law relationship, the value of Q (that is the same in equations (3) and (4)) allows one to evaluate how the infilled population relates to the original population. As an example, in Fig. 2 the distributions of $\sigma^2 + \overline{m}^2$ versus $\overline{m}$of two different synthetic populations are shown. These populations are "infilled" with a random criterion (i.e. at each infilling step the infilled cleavage is randomly selected). In the first case, α of equation (2) randomly spans from 0 to 1 (Fig. 2b), in the second case α = constant = 0.5, i.e. the newly developed cleavage locates exactly in the middle of the old cleavages (Fig. 2c). Regardless of the initial distribution, Q values obtained by equation (4) are about 0.35 and 0.5 in the first and second case, respectively. When $\alpha = 0.5$, the term $\alpha(1-\alpha)$ of equation (3) can be extracted from the summation and $2\alpha(1-\alpha) = 0.5$ by definition. The fact that $Q = 0.5$ confirms that $(\sum_{in=1}^{f} m_{in}^2)/f \approx (\sum_{l=1}^{n} m_l^2)/n$, that is the infilled population has the same statistical attributes of the original population (i.e. the infilled cleavage is randomly selected). When α = random, the term $\alpha(1-\alpha)$ cannot be extracted from the summation. Its average value when it randomly spans from 0 to 1 is 0.16666. Accordingly, to a first approximation we expect a Q value of about 0.33, very close to what we observe in the synthetic populations.

It is important to note that equation (4) does not include the strain, and allows evaluation of whether a give set of cleavage populations follows an infilling dominated evolution, by analysing only the average value and variance of cleavage spacing (and/or spacing to bed thickness ratio). This is possible only if we assume that the influence of other parameters (like rock composition) is negligible.

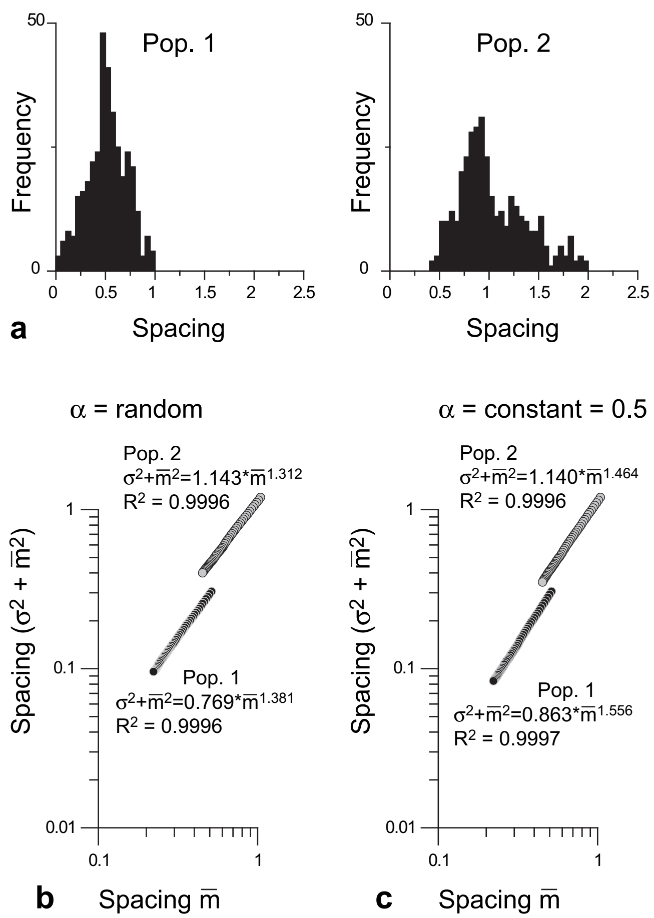

Fig. 2. (a) Synthetic populations spacing frequency distribution. (b) Evolution associated with a random infilling with α = random of the two synthetic populations. (c) Evolution associated with a random infilling with $\alpha = 0.5$ of the two synthetic populations. In (b) and (c), in the Y-axis is plotted the sum of spacing variance (σ^2) and square of spacing average value ($\overline{m}^2$), in the X-axis plotted the spacing average value ($\overline{m}$).

2.2. Spacing reduction by dissolution

In the hypothesis that spacing reduction is dominated by dissolution of material along the cleavage surfaces and both infilling and deformation outside cleavages are negligible, when a strain ε is applied (in the direction perpendicular to the cleavage surfaces) to a given population P_i, cleavage spacing reduces and P_i transforms in P_j. It can be roughly assumed that, in this hypothesis, the average value of P_j relates to that of P_i trough the following equation:

$$\overline{m}_j = (1 - \varepsilon_{ij})\overline{m}_i \qquad (5)$$

In order to compute the absolute amount of shortening of a given population, we should know the spacing distribution in the embryonic stage of cleavage evolution. When this information is not available, we can analyse only the relative shortening distribution (ε_{ij}) between different outcrops (i and j), which provides an indirect underestimated image of the absolute shortening distribution.

3. Data

Pressure solution cleavage data were collected in the Umbria–Marche carbonate multilayer exposed in the Sibillini thrust sheet of the Northern Apennines (Italy) (Fig. 3a). The exposed stratigraphic succession consists of three major mechanical units: (1) lower Jurassic, poorly layered platform limestones (*Calcare Massiccio* formation); (2) a lower Jurassic–Miocene pelagic sequence (where we collected cleavage data) including mainly well bedded limestones, marly limestones and marls (form *Corniola* to *Bisciaro* formations); (3) Miocene–Pliocene siliciclastic sequence consisting of mainly sandstones, clays and marls (e.g. Tavani et al., 2008). In the study area, the anticline associated with the Sibillini thrust has an NNW–SSE axial trend and is characterised by a constantly dipping backlimb (dip is about 30°), a smoothed transition to a wide flat-lying crestal sector, and a forelimb that includes a large overturned sector.

Distances between tectonic pressure solution cleavages were measured in 113-georeferenced field sites located along the backlimb, the crest and the forelimb (Fig. 3a). Most of the cleavage surfaces show a high angle to bedding and strike about parallel to the fold axis (Fig. 3b). In each outcrop, data acquisition was random, in order to reduce the bias between the cleavage population and its sampled part. Pressure solution cleavages at low angle to bedding (i.e. <35° and >145°; Fig. 3b) occur only in the forelimb of the anticline and were not included in the analysed dataset. In the studied anticline, cleavages are mostly in the fold limbs and their frequency progressively reduces toward the crestal sector, where cleavage becomes rare. This, coupled with the observation that cleavages strike about parallel to the fold axis (longitudinal

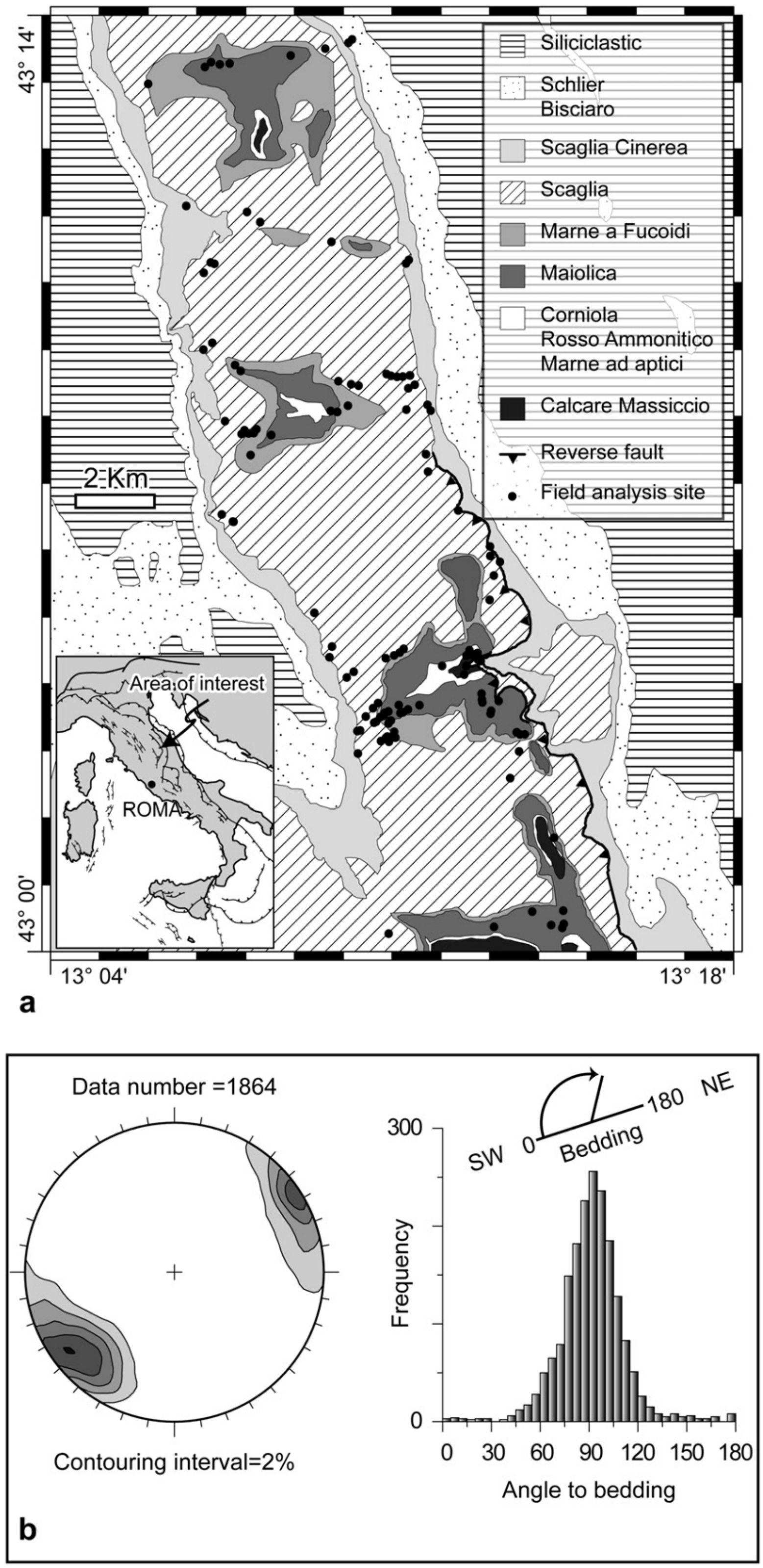

Fig. 3. (a) Geological map of the Sibillini thrust sheet with field sites distribution. (b) Contouring of poles to solution cleavage and frequency distribution of cleavage angle to bedding, data from the Jurassic-Miocene part of the multilayer.

cleavage), suggests that they developed during folding (e.g. Tavani et al., 2006), thus offering the possibility to study outcrops affected by different deformation intensities (Fig. 4) where cleavage developed almost synchronously and at rather constant depth. Pressure solution cleavages display an angular difference of about 15–20° with longitudinal joints, mostly located in the crestal sector. This ruled out any possible influence of joints on pressure solution cleavage occurrence and frequency.

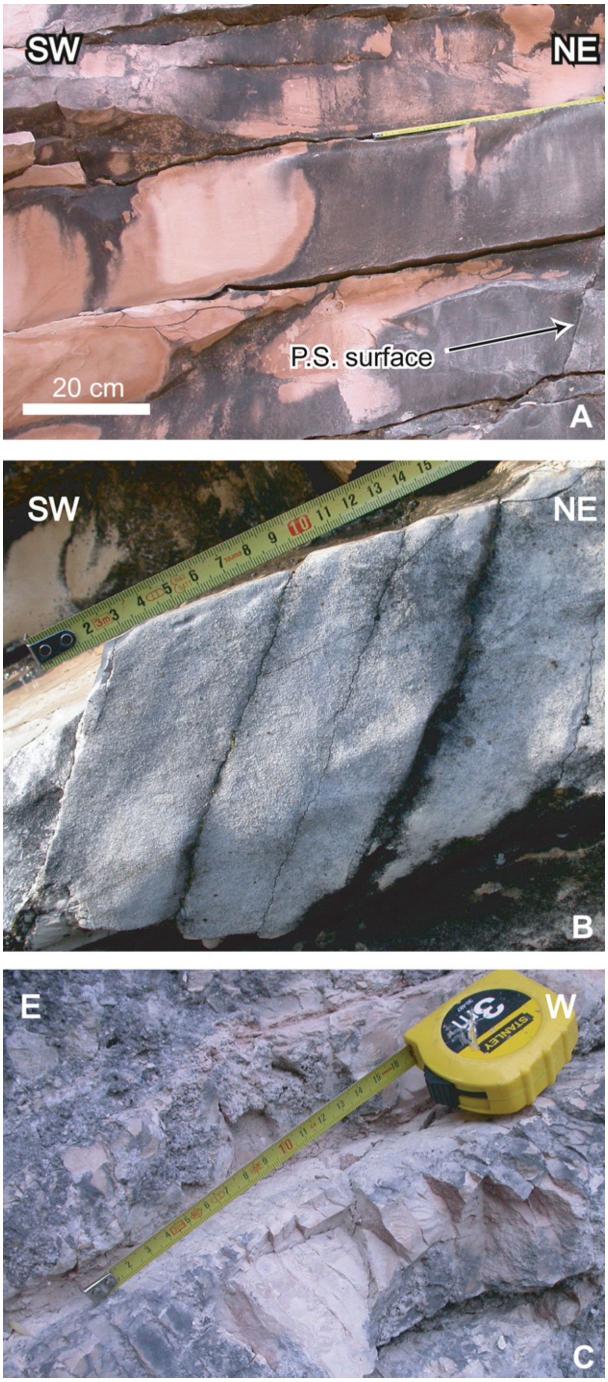

Fig. 4. Stratabound pressure solution cleavage populations with different degrees of deformation. Scaglia Rossa Formation.

Data were acquired and analysed in different formations to account for the well-known relationships between the deformation pattern and stratigraphical parameters (e.g. Marshak and Engelder, 1985; Corbett et al., 1987; Woodward and Rutherford, 1989; Protzman and Mitra, 1990; Gross, 1995; Couzens and Wiltschko, 1996; Fischer and Jackson, 1999; Chester, 2003; Peacock and Azzam, 2006; Tavani et al., 2008). The dataset mostly includes data collected in the Maiolica Formation (limestone), in the Scaglia Bianca Formation (marly limestone), and in the Scaglia Rossa Formation (marly limestone). In particular, the Maiolica and Scaglia Bianca formations are characterised by rather constant clay content compared with the Scaglia Rossa formation, which displays a higher lithological variability (even if always in the range of a marly limestone). Pressure solution cleavage, in these formations, is stratabound and, in many places, is associated with second order small veins oriented perpendicular to both bedding and pressure solution cleavage. At places, small-scale conjugate faults are also associated with cleavage surfaces and define a strike slip conjugate system bisected by cleavage-related veins. Cleavage, veins and small-scale faults define a system consistent with a stress field whose intermediate axis (σ_2) is oriented at high angle to bedding. This ensures that, during deformation, bed thickness is preserved.

Scatterograms of the entire dataset in the normal and bi-logarithmic spaces of cleavage spacing (*S*, measured perpendicularly to the cleavage surface) versus bed thickness (*H*) show that, in all the lithologies, the data dispersion is rather high (Fig. 5). The aim of these scatterograms is not that of defining a quantitative relationship between *S* and *H* (as spacing is also a function of lithological/environmental parameters and strain) but that of evaluating if a first–order correlation between *S* and *H* exists. To achieve this purpose we have used the Pearson correlation coefficient, which ranges from 0 (variables are independent) to 1 (perfect linear dependence). When applied to the *S* and *H* distribution, in the three analysed lithologies, it displays values of about 0.5, indicating that spacing is not independent of the bed thickness. The limited compositional variability, particularly in the Maiolica and Scaglia Bianca formations, and the collection of hundreds of data in several field sites, allows assuming that in the sampled population both compositional heterogeneities and strain distributions are independent of the bed thickness, i.e. it can be assumed that the distribution of strain and lithology in different bed thickness intervals is the same. This indicates that the first-order dependence of pressure solution cleavage spacing on bed thickness is not a statistical artefact.

3.1. Testing the infilling hypothesis

Equation (4) is applied to both *S* and *S/H* distributions of the three lithologies (Fig. 6) to obtain *Q* values (defined in Section 3.1), under the assumption that the "lithological bias" is limited. The obtained values of *Q* (i.e. the exponent of the power-law fit function − 1) range from about 1 to about 1.2. The high values of R^2 observed in the regression functions are mostly due to the term $(\overline{m})^2$. On the other hand, also the variance (σ^2) of both *S* and *S/H* displays a good correspondence with $\overline{m}$. The values of *Q* for the three lithologies are two/three times higher than those expected in a random infilling process, where *Q* spans from 0.33 to 0.5 (Fig. 2). Accordingly, in all the lithologies and for both *S* and *S/H*, we can approximately assume that, in equation (3), $(\sum_{in=1}^{f} m_{in}^2)/f \approx 2.5(\sum_{l=1}^{n} m_l^2)/n$, i.e. the population undergoing infilling is that with higher *S* and *S/H* values.

Both spacing and spacing to bed thickness ratio distributions are consistent with an infilling dominated evolution. On the other hand, only one of them can define the cleavage evolutionary pathway. This is because, in each outcrop, only one parameter (*S* or *S/H*) is representative of the deformation degree (the other is biased by the *H* values of the sampled population). If *S* was the representative parameter, the average value of *S/H* and *H* should display (with a large variability) a relationship. On the contrary, if *S/H* was the representative parameter, a relationship should be observed between *S* and *H*. Fig. 7 shows that the second hypothesis is more consistent. The data scatter is very high, mostly because in the same *H* range we are incorporating populations with very different degrees of deformation. Power law fitting shows that both exponent and R^2 associated with *S/H* are closer to zero than those associated with *S*, indicating that if a relationship can be assumed, it is between *S* and *H*, i.e. *S/H* is the driving parameter.

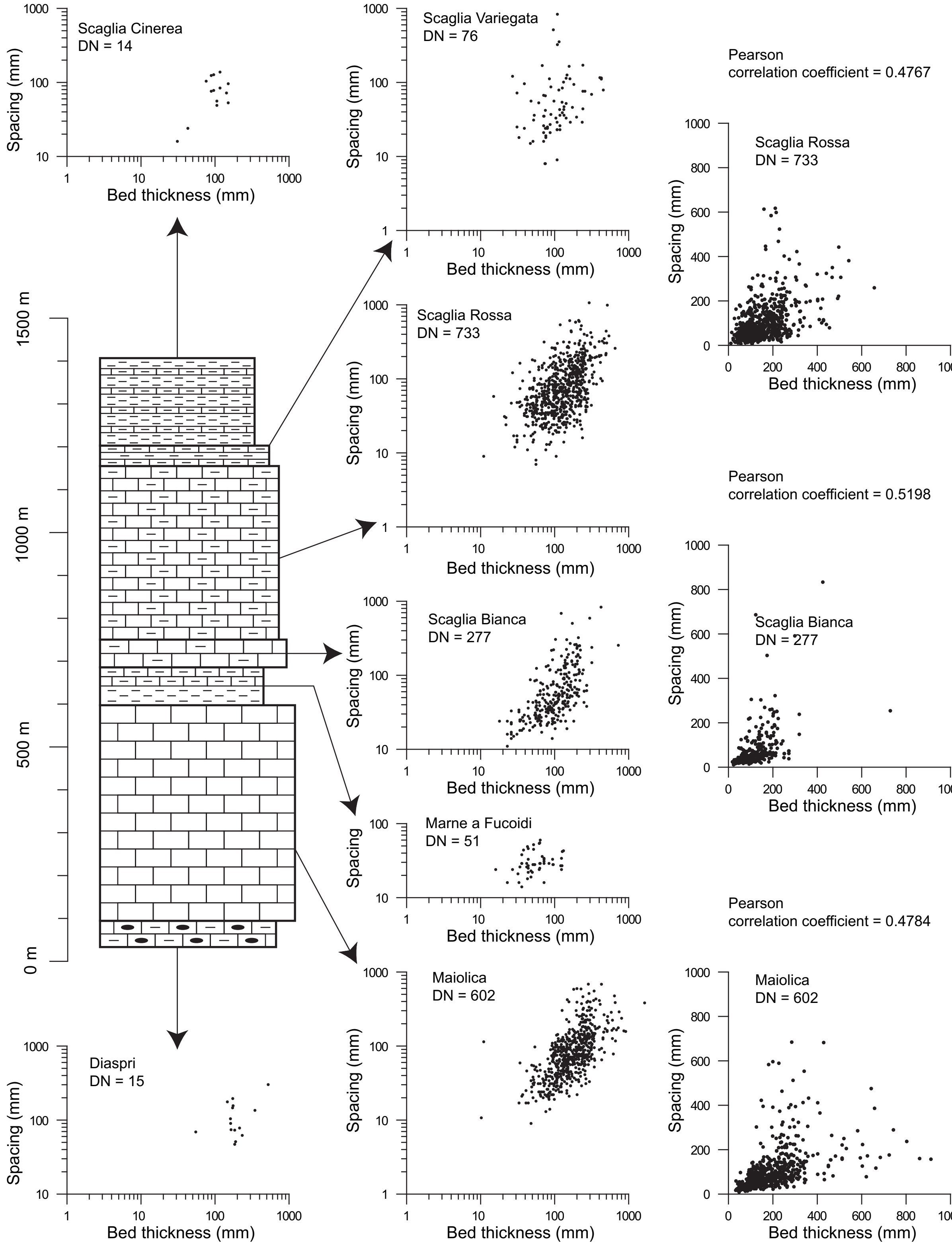

Fig. 5. Scatterograms, in the bi-logarithmic space, of cleavage spacing versus bed thickness. For the Scaglia Rossa, Scaglia Bianca and Maiolica formations are also shown the distribution in the normal space. In these formations, the Pearson's correlation coefficient between S and H is given. In all the plots DN refers to the number of cleavages collected in the corresponding unit.

3.2. Testing the dissolution hypothesis

Application of equation (5) to our dataset provides the relative shortening distribution shown in Fig. 8. In all the lithologies, the average value of ε_{ij} is about 50%; with a significant amount of data higher than 70%. These values are strongly higher than those observed in nature, which commonly range between 5 and 40% (e.g. Engelder and Engelder, 1977; Hudleston and Holst, 1984; Ferrill

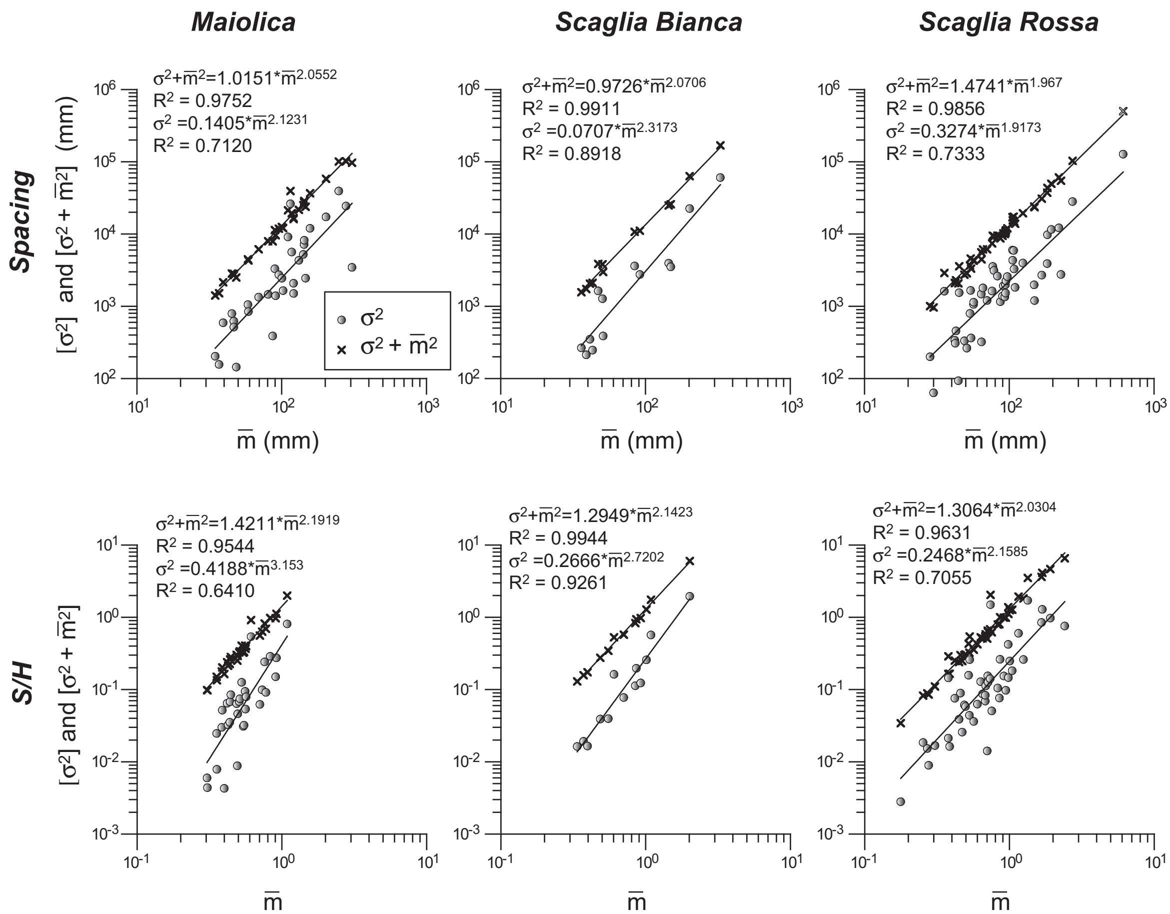

Fig. 6. Scatterograms of variance + square of average value [$\sigma^2 + \overline{m}^2$] and variance [$\sigma^2$] versus average value [$\overline{m}$] of both *S* and *S/H* of different outcrops. See text for details.

and Dunne, 1989; Protzman and Mitra, 1990; Holl and Anastasio, 1995; Evans et al., 2003) and only rarely exceed 40% (e.g. Alvarez et al., 1978). This, coupled with the observation that the values of ε_{ij} obtained by equation (5) provide underestimated values of the strain, lead to the conclusion that cleavage spacing distributions in the study area cannot be interpreted as the result of mere dissolution without infilling.

4. Discussion

Spacing distributions in our dataset are consistent with an evolution of pressure solution cleavage mostly achieved by infilling, i.e. a cleavage population progressively modifies the spacing by the development of new dissolution surfaces into the microlithons. At each infilling step, the infilled population

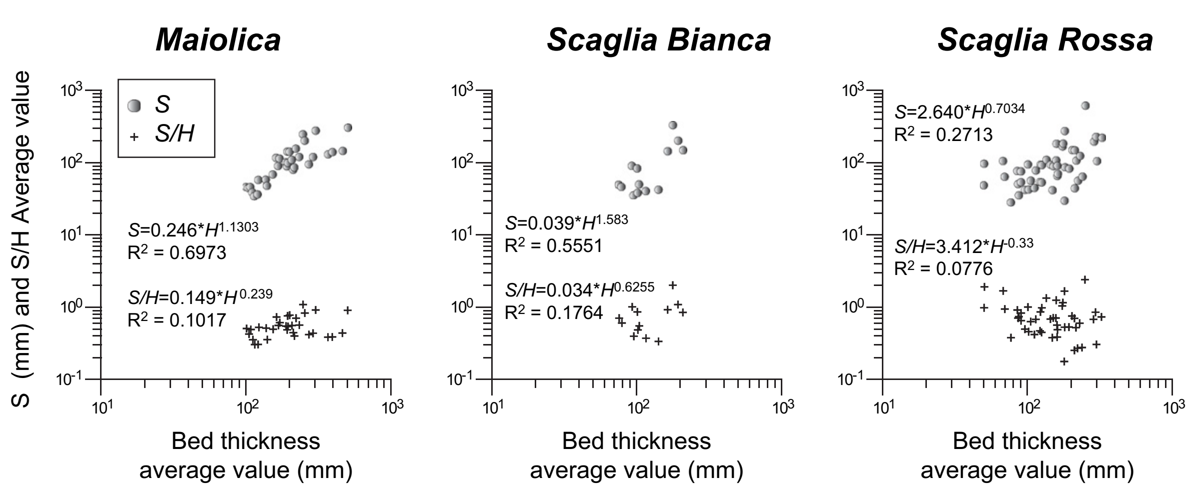

Fig. 7. Relationships between average value of both *S* and *S/H*, and average value of bed thickness in different outcrops.

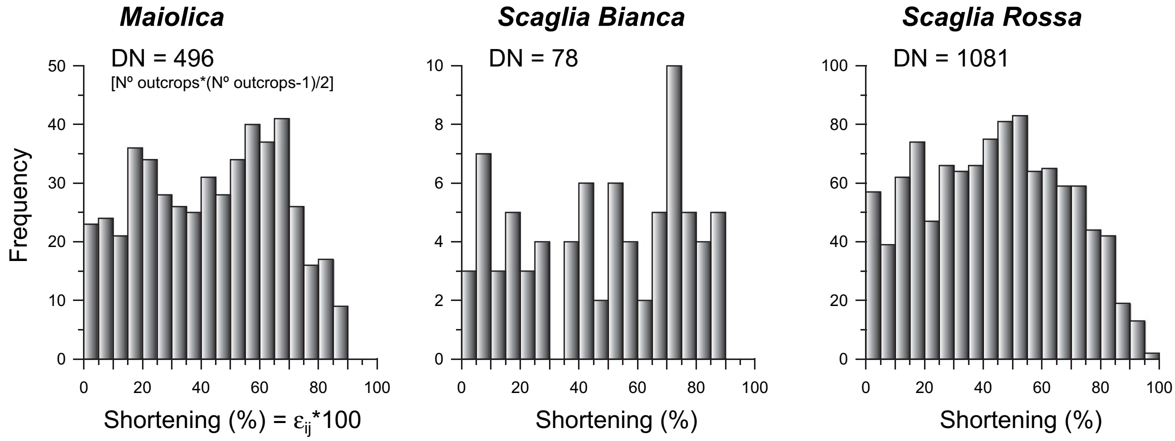

Fig. 8. Frequency distribution of ε_{ij} (displayed as shortening = 100 × ε_{ij}). DN refers to the number of observations of ε_{ij}, which coincides with: [numbers of outcrops] × [numbers of outcrops − 1]/2. See text for details.

includes cleavages with the higher spacing to bed thickness ratios. *Q* values computed in the Umbria–Marche carbonate multilayer are based on the assumption that infilling was the only acting mechanism during cleavage development. This implies a strong oversimplification of the cleavage evolutionary pathway, which includes a contribution of spacing reduction by progressive dissolution. On the other hand, for the three studied lithologies, *Q* values are two to three times greater than those expected in a random infilling, thus strongly supporting the idea that the infilling process and, consequently, the spacing distribution in the studied cleavage populations are not the result of a random process. This evidence for a non-random process contrasts with the results of Railsback (1998), whose analysis using the ratio of standard deviation to mean of cleavage spacing found little evidence for self-organization. Such a ratio, however, provides a static evaluation of the population organization and does not allow evaluating the incremental organization of pressure solution cleavage. A direct consequence of the dependence of the infilling on the S/H ratio is that the cleavage spacing population during infilling tends to values scaling with bed thickness and/or preserves this scaling relationship. The relationships between *S/H* variance and average value do not display significant differences at different average values and this suggests that the deformation mechanisms operating in the mature evolutionary stages were the same as the ones acting in the early stages.

In our cross-sectional analyses, we cannot discriminate between apparent infilling associated with the lateral propagation of pre-existing cleavages, and infilling *sensu strictu* by newly developed cleavages. The lack of evident discontinuities in the evolutionary pathways of Fig. 6, suggests that both processes depend on the cleavage *S/H* ratio, i.e. lateral propagation is faster between cleavages with high S/H ratios and new cleavages preferentially develop between old cleavages with high *S/H* ratios.

Fletcher and Pollard (1981) proposed an elastic anticrack model for pressure solution cleavage where deformation is everywhere elastic, except along the cleavage surface. The maximum acting stress concentrates around the cleavage tip, thus allowing cleavage lateral propagation, and relaxes around the cleavage surface. This relaxation area idea is “imported” from the vast literature on jointing, where it scales with the joint height. It is referred to as the stress reduction shadow and it is commonly invoked to explain the observed correlation between joint spacing and layer thickness (e.g. Cox, 1952; Lachenbruch, 1961; Pollard and Segall, 1987; Narr and Suppe, 1991; Fischer et al., 1995; Gross et al., 1995; Becker and Gross, 1996; Bai and Pollard, 2000; among others). Within the stress reduction shadow, the formation of new joints is inhibited by decreasing the module of the acting stress. The shape and magnitude of stress reduction shadows depend on joint height, displacement profile along joints, and mechanical properties of the rock. Application of this concept to pressure solution cleavage, however, is influenced by two main points: (1) Joints are assumed to act as free surfaces of zero stress, whereas the same assumption does not hold for dissolution surfaces; (2) a necessary condition to obtain a stress reduction shadow scaling with the bed thickness is that, in a cross-sectional view, displacement decreases toward the cleavage tip/tips, where it is eventually zero. This condition is satisfied for intrabed pressure solution cleavage. When the cleavage surfaces reach the layer boundaries, maintaining a constant shadow requires constant displacement increments through the cleavage cross-sectional profile. Otherwise, displacement gradients by preferential dissolution in the tip region cause the pressure reduction shadow to progressively disappear.

Accordingly, we expect that stress reduction shadows explaining the scaling relationship between spacing and bed thickness are quite weak and, consequently, they slightly influence the development and propagation of pressure solution cleavages. A consequence of this is the higher scatter of stratabound pressure solution cleavage *S* and *H* data.

5. Conclusions

We have developed quantitative predictive relationships to investigate, from natural datasets, whether pressure solution cleavage spacing evolves by dominant dissolution along the same surfaces, or by episodic infilling by newly formed cleavage surfaces. Application to data collected in the Sibillini thrust sheet of the Northern Apennines provided results consistent with an evolution of pressure solution cleavage, mostly achieved by infilling, were new dissolution surfaces preferentially developed between old cleavages characterised by high spacing to bed thickness ratios. As a result, cleavage spacing values scale with the thickness of the layer. Our analyses suggest that the anticrack behavior of Fletcher and Pollard (1981) can be broadened, by including a pressure reduction shadow around the cleavage, able to provide a mechanical explanation to our statistical observations.

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Appendix. (see Table 1 for variables definition)

Equation 1

Let us assume a cleavage population P_i consisting of n elements, where each cleavage is characterised by a value of m (being m either S or S/H). When f elements of the initial population are infilled, P_i transforms in P_j. Each m_{in} value of the infilled population is replaced by two m values (m'_{in} and m''_{in}), which relate to m_{in} through the following equations:

$$m'_{in} = \alpha_{in} m_{in} \tag{A 1.1}$$

$$m''_{in} = (1-\alpha_{in}) m_{in} \tag{A 1.2}$$

being $0 \le \alpha_{in} \le 1$

the average value of m in the P_i population is:

$$\overline{m}_i = \frac{\sum_{l=1}^{n} m_l}{n} \tag{A 1.3}$$

the average value of m in the P_j population is (A 1.4):

$$\begin{aligned}\overline{m}_j &= \frac{\sum_{l=1}^{n-f} m_l + \sum_{in=1}^{f}(m'_{\mathrm{in}} + m''_{\mathrm{in}})}{n+f} \\ &= \frac{\sum_{l=1}^{n-f} m_l + \sum_{in=1}^{f}(1-\alpha_{in}+\alpha_{in})m_{in}}{n+f} = \frac{\sum_{l=1}^{n} m_l}{n+f} \\ &= \overline{m}_i \frac{n}{n+f}\end{aligned} \tag{A 1.4}$$

Equation 2

The variance of m in the P_i population is (A 2.1):

$$\sigma_i^2 = \frac{\sum_{l=1}^{n}(m_l-\overline{m}_i)^2}{n} = \frac{\sum_{l=1}^{n}\left[m_l^2+\overline{m}_i^2-2(m_l\overline{m}_i)\right]}{n}$$
$$= \frac{\sum_{l=1}^{n} m_l^2}{n} + \frac{n\overline{m}_i^2}{n} - 2\overline{m}_i\frac{\sum_{l=1}^{n} m_l}{n} = \frac{\sum_{l=1}^{n} m_l^2}{n} - \overline{m}_i^2 \tag{A 2.1}$$

The variance of m in the P_j population is (A 2.2):

$$\begin{aligned}\sigma_j^2 &= \frac{\sum_{l=1}^{n-f} m_l^2 + \sum_{\mathrm{in}=1}^{f}\left[(m'_{\mathrm{in}})^2+(m''_{\mathrm{in}})^2\right]}{n+f} - \overline{m}_j^2 \\ &= \frac{\sum_{l=1}^{n} m_l^2 + \sum_{\mathrm{in}=1}^{f}\left[(m'_{\mathrm{in}})^2+(m''_{\mathrm{in}})^2\right] - \sum_{\mathrm{in}=1}^{f}(m_{\mathrm{in}})^2}{n+f} - \overline{m}_j^2 \\ &= \frac{\sum_{l=1}^{n} m_l^2 + \sum_{\mathrm{in}=1}^{f}\left[(m'_{\mathrm{in}})^2+(m''_{\mathrm{in}})^2\right] - \sum_{\mathrm{in}=1}^{f}(m'_{\mathrm{in}}+m''_{\mathrm{in}})^2}{n+f} \\ &\quad - \overline{m}_j^2 \\ &= \frac{\sum_{l=1}^{n} m_l^2 - 2\sum_{\mathrm{in}=1}^{f} m'_{\mathrm{in}} m''_{\mathrm{in}}}{n+f} - \overline{m}_j^2\end{aligned} \tag{A 2.2}$$

The difference between σ_i^2 and σ_j^2 is (A 2.3):

$$\begin{aligned}\Delta\sigma^2 &= \frac{\sum_{l=1}^{n} m_l^2}{n} - \overline{m}_i^2 \\ &\quad - \left[\frac{\sum_{l=1}^{n} m_l^2 - 2\left(\sum_{in=1}^{f}\alpha_{\mathrm{in}}(1-\alpha_{\mathrm{in}})m_{\mathrm{in}}^2\right)}{n+f} - \overline{m}_j^2\right] \\ &= \frac{\left(\sum_{l=1}^{n} m_l^2\right)(n+f-n)}{n(n+f)} - \overline{m}_i^2\frac{\left[(n+f)^2-n^2\right]}{(n+f)^2} \\ &\quad + \frac{2\left(\sum_{in=1}^{f}\alpha_{\mathrm{in}}(1-\alpha_{\mathrm{in}})m_{\mathrm{in}}^2\right)}{n+f}\end{aligned} \tag{A 2.3}$$

The difference between $\overline{m}_i$ and $\overline{m}_j$ is (A 2.4):

$$\Delta\overline{m} = \overline{m}_i - \overline{m}_i\frac{n}{n+f} = \overline{m}_i\frac{f}{n+f} \tag{A 2.4}$$

The ratio between equations (A 2.3) and (A 2.4) is (A 2.5):

$$\begin{aligned}\frac{\Delta\sigma^2}{\Delta\overline{m}} &= \frac{\sum_{l=1}^{n} m_l^2}{n\overline{m}_i} - \overline{m}_i\frac{[f+2\mathrm{n}]}{(n+f)} + \frac{2\left(\sum_{in=1}^{f}\alpha_{in}(1-\alpha_{in})m_{in}^2\right)}{f\overline{m}_i} \\ &= \frac{\sigma_i^2+\overline{m}_i^2}{\overline{m}_i} - \frac{\overline{m}_i^2}{\overline{m}_i} - \overline{m}_i\frac{n}{(n+f)} + \frac{2\left(\sum_{in=1}^{f}\alpha_{in}(1-\alpha_{in})m_{in}^2\right)}{f\overline{m}_i} \\ &= \frac{\sigma_i^2 - \overline{m}_i^2[n/(n+f)] + 2\left(\sum_{in=1}^{f}\alpha_{in}(1-\alpha_{in})m_{in}^2\right)\Big/ f}{\overline{m}_i}\end{aligned} \tag{A 2.5}$$

From equations (A 2.4) and (A 2.5) derive that (A 2.6):

$$\begin{aligned}\frac{\partial\sigma^2}{\partial\overline{m}} &= \lim_{\Delta\overline{m}\to 0}\frac{\Delta\sigma^2}{\Delta\overline{m}} = \lim_{\frac{f}{n+f}\to 0}\frac{\Delta\sigma^2}{\Delta\overline{m}} = \lim_{\frac{n}{n+f}\to 1}\frac{\Delta\sigma^2}{\Delta\overline{m}} \\ &= \frac{\sigma^2}{\overline{m}} + \frac{+2\left(\sum_{in=1}^{f}\alpha_{in}(1-\alpha_{in})m_{in}^2\right)\Big/ f - \overline{m}^2}{\overline{m}}\end{aligned} \tag{A 2.6}$$

By combining equation (A 2.6) with equation (4) ($Q = \frac{2(\sum_{in=1}^{f}\alpha_{in}(1-\alpha_{in})m_{in}^2)}{f} / \frac{\sum_{l=1}^{n} m_l^2}{n}$), we obtain (A 2.7):

$$\begin{aligned}\frac{\partial\sigma^2}{\partial\overline{m}} &= \frac{\sigma^2}{\overline{m}} + \frac{\left(Q\sum_{l=1}^{n} m_l^2\right)\Big/ n - \overline{m}^2}{\overline{m}} = \frac{\sigma^2}{\overline{m}} - \overline{m} + Q\frac{\sigma^2}{\overline{m}} + Q\overline{m} \\ &= \frac{\sigma^2}{\overline{m}}(1+Q) - \overline{m}(1-Q)\end{aligned} \tag{A 2.7}$$

The solution for this equation is provided by equation (5):

$$\sigma^2 = k(\overline{m})^{Q+1} - \overline{m}^2.$$

References

Alvarez, W., Engelder, T., Geiser, P., 1978. Classification of solution cleavage in pelagic limestones. Geology 6, 263–266.

Andrews, L.A., Railsback, L.B., 1997. Controls on stylolite development: morphologic, lithologic and temporal evidence from bedding-parallel and transverse stylolites from the U.S. Appalachians. Journal of Geology 105, 59–73.

Arthaud, F., Mattauer, M., 1969. Exemples de stylolites d'origine tectonique dans le Languedoc, leurs relations avec la tectonique cassante. Bulletin de la Societé Géologique de France 11, 738–744.
Bai, T., Pollard, D.D., 2000. Fracture spacing in layered rocks: a new explanation based on the stress transition. Journal of Structural Geology 22, 43–57.
Becker, A., Gross, M.R., 1996. Mechanism for joint saturation in mechanically layered rocks: an example from southern Israel. Tectonophysics 257, 223–237.
Carannante, G., Guzzetta, G., 1972. Stiloliti e sliccoliti come meccanismo di deformazione delle masse rocciose. Bollettino della Società dei Naturalisti in Napoli 81, 157–170.
Chester, J.S., 2003. Mechanical stratigraphy and fault–fold interaction, Absaroka thrust sheet, Salt River Range, Wyoming. Journal of Structural Geology 25, 1171–1192.
Corbett, K., Friedmean, M., Spang, J., 1987. Fracture development and mechanical stratigraphy of Austin Chalk, Texas. AAPG Bulletin 71, 17–28.
Couzens, B.A., Wiltschko, D.V., 1996. The control of mechanical stratigraphy on the formation of triangle zones. Bulletin of Canadian Petroleum Geology 44, 165–179.
Cox, H.L., 1952. The elasticity and strength of paper and other fibrous materials. British Journal of Applied Physics 3, 72–79.
Dieterich, J.H., 1969. Origin of cleavage in folded rocks. American Journal of Science 267, 155–165.
Dunnington, H.V., 1954. Stylolite development post-dates rock induration. Journal of Sedimentary Petrology 24, 27–49.
Durney, D.W., Kisch, H.J., 1994. A field classification and intensity scale for first-generation cleavages. Journal of Australian Geology and Geophysics 15, 257–295.
Engelder, T., Engelder, R., 1977. Fossil distortion and decollement tectonics of the Appalachian Plateau. Geology 5, 457–460.
Evans, M.A., Lewchuk, M.T., Elmore, R.D., 2003. Strain partitioning of deformation mechanisms in limestones: examining the relationship of strain and anisotropy of magnetic susceptibility (AMS). Journal of Structural Geology 25, 1525–1549.
Ferrill, D.A., Dunne, W.M., 1989. Cover deformation above a blind duplex: an example from West Virginia, U.S.A. Journal of Structural Geology 11, 421–431.
Fischer, M.B., Jackson, P.B., 1999. Stratigraphic controls on deformation patterns in fault-related folds: a detachment fold example from the Sierra Madre Oriental, northeast Mexico. Journal of Structural Geology 21, 613–633.
Fischer, M., Gross, M.R., Engelder, T., Greenfield, R.J., 1995. Finite element analysis of the stress distribution around a pressurized crack in a layered elastic medium: implications for the spacing of fluid-driven joints in bedded sedimentary rock. Tectonophysics 247, 49–64.
Fletcher, R.C., Pollard, D.D., 1981. Anticrack model for pressure solution surfaces. Geology 9, 419–424.
Fueten, F., Robin, P.Y.F., Schweinberger, M.M., 2002. Finite element modelling of the evolution of pressure solution cleavage. Journal of Structural Geology 24, 1055–1064.
Groshong, R.H., 1988. Low-temperature deformation mechanisms and their interpretation. Geological Society of America Bulletin 100, 1329–1360.
Gross, M.R., Fischer, M.P., Engelder, T., Greenfield, R.J., 1995. Factors controlling joint spacing in interbedded sedimentary rocks: integrating numerical models with field observations from the Monterey Formation, USA. In: Ameen, M.S. (Ed.), Fractography: Fracture Topography as a Tool in Fracture Mechanics and Stress Analysis. Geological Society, London, Special Publication, vol. 92, pp. 215–233.
Gross, M.R., 1995. Fracture partitioning: failure mode as a function of lithology in the Monterey Formation of coastal California. Geological Society of America Bulletin 107, 779–792.
Holl, J.E., Anastasio, D.J., 1995. Cleavage development within a foreland fold and thrust belt, southern Pyrenees, Spain. Journal of Structural Geology 17, 357–369.
Hudleston, P.J., Holst, T.B., 1984. Strain analysis and fold shape in a limestone layer and implications for layer rheology. Tectonophysics 106, 321–347.
Illies, H., 1975. Intraplate tectonics in stable Europe as related to plate tectonics in the Alpine system. Geologische Rundschau 64, 677–699.
Koehn, D., Renard, F., Toussaint, R., Passchier, C.W., 2007. Growth of stylolite teeth patterns depending on normal stress and finite compaction. Earth and Planetary Science Letters 257, 582–595.
Lachenbruch, A.H., 1961. Depth and spacing of tension cracks. Journal of Geophysical Research 66, 4273–4292.
Marshak, S., Engelder, T., 1985. Development of cleavage in limestones of a fold-trust belt in eastern New York. Journal of Structural Geology 7, 345–359.
Merino, E., Ortoleva, P., Strickholm, P., 1983. Generation of evenly spaced pressure-solution seams during (late) diagenesis. Contributions to Mineralogy and Petrology 82, 360–370.
Mitra, G., Yonkee, W.A., 1985. Relationship of spaced cleavage to folds and thrusts in the Idaho–Utah–Wyoming thrust belt. Journal of Structural Geology 7, 361–373.
Narr, W., Suppe, J., 1991. Joint spacing in sedimentary rocks. Journal of Structural Geology 13, 1037–1048.
Ohlmacher, G.C., Aydin, A., 1995. Progressive deformation and fracture patterns during foreland thrusting in the southern Appalachians. American Journal of Science 295, 943–987.
Park, W.C., Schot, E.H., 1968. Stylolites: their origin and nature. Journal of Sedimentary Petrology 38, 175–191.
Peacock, D.C.P., Azzam, I.N., 2006. Development and scaling relationships of a stylolite population. Journal of Structural Geology 28, 1883–1889.
Pollard, D.D., Segall, R.H., 1987. Theoretical displacements and stresses near fractures in rock: with application to faults, joints, veins, dikes, and solution surfaces. In: Atkinson, B.K. (Ed.), Fracture Mechanics of Rock. Academic Press, New York, pp. 227–349.
Protzman, G.M., Mitra, G., 1990. Strain fabric associated with the Meade thrust sheet: implications for cross-section balancing. Journal of Structural Geology 12, 403–417.
Railsback, L.B., Andrews, L.M., 1995. Tectonic stylolites in the "undeformed" Cumberland Plateau of southern Tennessee. Journal of Structural Geology 17, 911–915.
Railsback, L.B., 1998. Evaluation of spacing of stylolites and its implications for self-organisation of pressure dissolution. Journal of Sedimentary Research 68, 2–7.
Rutter, E.H., 1976. The kinetics of rock deformation by pressure solution. Philosophical Transactions of the Royal Society of London 283, 203–219.
Rutter, E.H., 1983. Pressure solution in nature, theory and experiment. Journal of the Geological Society 140, 725–740.
Sans, M., Vergés, J., Gomis, E., Parés, J.M., Schiattarella, M., Travé, A., Calvet, F., Santanach, P., Doulcet, A., 2003. Layer parallel shortening in salt-detached folds: constraint on cross-section restoration. Tectonophysics 372, 85–104.
Siddans, A.W.B., 1972. Slaty cleavage–a review of research since 1815. Earth Science Review 8, 205–232.
Stockdale, P.B., 1922. Stylolites; Their Nature and Origin, vol. 9. Indiana University Studies. 1–97.
Tavani, S., Storti, F., Fernández, O., Mu ñoz, J.A., Salvini, F., 2006. 3-D deformation pattern analysis and evolution of the Añisclo anticline, southern Pyrenees. Journal of Structural Geology 28, 695–712.
Tavani, S., Storti, F., Salvini, F., Toscano, C., 2008. Stratigraphic versus structural control on the deformation pattern associated with the evolution of the Mt. Catria anticline, Italy. Journal of Structural Geology 30, 664–681.
Weyl, P.K., 1959. Pressure solution and the force of crystallization – a phenomenological theory. Journal of Geophysical Research 64, 2001–2025.
Woodward, N.B., Rutherford Jr., E., 1989. Structural lithic units in external orogenic zones. Tectonophysics 158, 247–258.